(d)

**• definition of the VaR and its use.**

Value at Risk (VaR) is the maximum possible loss of an asset or portfolio of assets at a certain quantile level within a given range. By definition, VaR is a function of two variables, one is the length of the time range and the other is the quantile level 1-α. Here, VaR is defined as a negative value, then under a quantile level 1-α, and VaR can be expressed as:

Where: is the return of the asset at time t; is the information set at time t-1; , is the weight and is the return of the asset i at time t in the portfolio (Zhang, Wei and Yu et al. 2014).

**• whether copula-based multivariate models perform well in the VaR estimation.**

Aas and Berg [13] pointed out that multivariate methods also have some drawbacks in predicting the portfolio's VaR. Some recent literature got improvement in the prediction of portfolio's VaR through using copula function, which could fully consider the nonlinear correlation of variables (Cheng, Li and Shi 2007). However, the traditional binary copula function cannot describe the relationship among multiple variables due to the Stability limitation, and cannot predict the VaR of portfolio when there are more than two variables in a portfolio.

**• the effects of misspecified marginals and copulas on the VaR estimation.**

The VaR estimates are very poor and suffer from computational problems when estimating GARCH models for small samples (Hwang and Valls Pereira, 2006). When the sample dimension increases, the biases in the volatility parameters are much smaller, whereas those in the copula parameters remain almost unchanged or even increase. In this case, copula misspecifications do play a role for VaR computation. However, these effects depend heavily on the sign of the dependence: if it is negative, the bias can be as large as 70%; if it is positive, the bias is much smaller, and the effects on quantile estimation are much more limited, if not completely offset by marginal misspecifications (Fantazzini 2009).

**• time-varying copula models for the VaR estimation**

VaR t (α) is the α-quantile of the conditional distribution of portfolio returns at time t. VaR is calculated for the true model at each point in time, and we compute the MSE between the estimated and true values in the same way as in, where K is the number of Monte Carlo replications, and  and  denote the true and estimated dependence paths at replication k, respectively (Manner and Reznikova, 2012).

Reference:

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